NONSTATIONARY THERMOCAPILLARY DRIFT OF A DROP OF VISCOUS LIQUID

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1. The mathematical formulation of the problem of the motion of a drop of viscous liquid under the action of thermocapillary forces consists of the following [1]. It is necessary to find a surface Γ_t, separating the space R^3 into a bounded singly connected region Ω^+_t and its complement $\Omega_t^- = R^3 \setminus \bar{\Omega}_t^+$, and the velocity field v, the pressure field p, and the temperature field T, which depend on the time t and the spatial coordinates $\mathbf{x} = (x_1, x_2, x_3)$ and satisfy the differential equations

$$\partial \mathbf{v}/\partial t + \mathbf{v}\cdot\nabla\mathbf{v} = -\rho^{-1}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{g},\ \nabla\cdot\mathbf{v} = 0,$$
$$\partial T/\partial t + \mathbf{v}\cdot\nabla T = \chi\nabla^2 T \text{ in } R^3\setminus\Gamma_t, \quad (1.1)$$

and the joining conditions

$$[P\cdot\mathbf{n}]^{+}_{-} = \sigma K\mathbf{n} + \nabla_\Gamma\sigma,\ V_n = \mathbf{v}\cdot\mathbf{n},\ [\mathbf{v}]^{+}_{-} = 0,$$
$$[\varkappa\partial T/\partial n]^{+}_{-} = 0,\ [T]^{+}_{-} = 0 \text{ on } \Gamma_t, \quad (1.2)$$

the conditions on infinity

$$\mathbf{v} \to 0 \quad \text{as } |\mathbf{x}| \to \infty \quad (1.3)$$

and the initial conditions

$$\mathbf{v} = \mathbf{v}_0,\ T = T_0,\ \Gamma_t = \Gamma_0 \quad \text{at} \quad t = 0. \quad (1.4)$$

Here the density ρ, the kinematic coefficient of viscosity ν, the coefficient of thermal diffusivity χ, and the coefficient of thermal conductivity ϰ are piecewise-constant with a surface of discontinuity Γ_t; the coefficient of surface tension σ is a known function of the temperature; $P = -pI + 2\mu D(\mathbf{v})$, stress tensor; μ = ρν, dynamic coefficient of viscosity; I, unit tensor; D(v), tensor of the deformation velocities, equal to the symmetric part of the tensor ∇v; V_n, velocity of Γ_t along the outer normal **n**, to Ω^+_t; K, sum of the principal curvatures Γ_t (the trace of the curvature tensor); ∇ and ∇_Γ, gradient operator in R^3 and gradient operator on Γ_t, respectively. The symbol $[\cdot]^{+}_{-}$ denotes a jump, i.e., $[f]^{+}_{-} = f^+ - f^-$, where $f^\pm$ are the limiting values of the function f(x, t) as x approaches a point on the surface Γ_t from $\Omega^\pm_t$. The mass-force density **g**(x, t), the functions $\mathbf{v}_0(\mathbf{x})$, $T_0(\mathbf{x})$, and the surface Γ_0 are given.

It is evident from the boundary conditions (1.2) that the velocity and temperature fields are continuous across Γ_t, while the pressure field and tangential stresses undergo a jump. As a result, in the presence of a temperature gradient there arise thermocapillary forces which, together with the bouyancy forces, cause the drop to drift. For simplicity, here we study the particular variant of the initial conditions $\mathbf{v}_0 = 0$, $T_0 = \mathbf{A}\cdot\mathbf{x}$, $\Gamma_0 = \{|\mathbf{x}| = a\}$. In addition, it is assumed that $\mathbf{A} = (0, 0, A)$ and $\mathbf{g} = (0, 0, g(t))$. This problem describes the acceleration of a drop by thermocapillary and buoyancy forces. The case of constant σ and g is studied in [2, 3].

2. We transform now to a noninertial coordinate system, fixed to the center of mass of the drop, moving in the starting system with the velocity $\mathbf{u}(t) = (0, 0, u(t))$, i.e.,

$$\mathbf{x}' = \mathbf{x} - \int_0^t \mathbf{u}(t)\,dt, \quad t' = t.$$

We introduce the new functions sought:

$$\mathbf{v}' = \mathbf{v} - \mathbf{u},\ p' = p + \rho\mathbf{x}[\mathbf{g} - d\mathbf{u}/dt],\ T' = T,$$

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in the primed variables the system of equations (1.1) and (1.2) then transforms into a system of the same form with $\mathbf{g}' = 0$, $V'_n = V_n - \mathbf{u}\cdot\mathbf{n}$, $P' = -[p' + \rho\mathbf{x}'(d\mathbf{u}/dt - \mathbf{g})]I + 2\mu D(\mathbf{v}')$.

Suppose that $\sigma(T) = \sigma_0 - \sigma_1 T$, where σ_0 and σ_1 are positive numbers. We select as the length, time, velocity, pressure, and temperature scales the quantities a, a^2/ν^-, $\sigma_1 Aa/\mu^-$, $\sigma_1 A$, and Aa. Then the equations of motion after dropping the primes assume the form

$$\begin{gathered} \partial\mathbf{v}/\partial t + \mathrm{Ma}\,\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p/\rho^0 + \nu^0\nabla^2\mathbf{v},\ \nabla\cdot\mathbf{v} = 0, \\ \mathrm{Pr}\,[\partial T/\partial t + \mathrm{Ma}\,\mathbf{v}\cdot\nabla T] = \chi^0\nabla^2 T \text{ in } \Omega_t^+, \\ \partial\mathbf{v}/\partial t + \mathrm{Ma}\,\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \nabla^2\mathbf{v},\ \nabla\cdot\mathbf{v} = 0, \\ \mathrm{Pr}\,[\partial T/\partial t + \mathrm{Ma}\,\mathbf{v}\cdot\nabla T] = \nabla^2 T \text{ in } \Omega_t^-; \end{gathered} \tag{2.1}$$

$$\{-p^+ + p^- + (\rho^0 - 1)(du/dt - \eta)x_3\}\mathbf{n} + 2\mu^0 D(\mathbf{v}^+)\cdot\mathbf{n} - 2D(\mathbf{v}^-)\cdot\mathbf{n} = (\mathrm{We}^{-1} - T)K\mathbf{n} - \nabla_\Gamma T, \tag{2.2}$$

$$\begin{gathered} V_n = \mathbf{v}^+\cdot\mathbf{n},\ V_n = \mathbf{v}^-\cdot\mathbf{n},\ \mathbf{v}^+\cdot\boldsymbol{\tau} = \mathbf{v}^-\cdot\boldsymbol{\tau}, \\ \varkappa^0\partial T^+/\partial n = \partial T^-/\partial n,\ T^+ = T^- \text{ on } \Gamma_t; \\ \mathbf{v} + \mathbf{u} \to 0 \quad \text{as} \quad |\mathbf{x}| \to \infty; \end{gathered} \tag{2.3}$$

$$\mathbf{v} = 0,\ \mathbf{u} = 0,\ T = x_3,\ \Gamma_t = \{|\mathbf{x}| = 1\} \quad \text{at} \quad t = 0. \tag{2.4}$$

Here $\boldsymbol{\tau}$ is the vector tangent to Γ_t; $\rho^0 = \rho^+/\rho^-$; $\nu^0 = \nu^+/\nu^-$; $\mu^0 = \rho^0\nu^0$; $\chi^0 = \chi^+/\chi^-$; $\varkappa^0 = \varkappa^+/\varkappa^-$; $\mathrm{Ma} = (\mu^-\nu^-)^{-1}\sigma_1 Aa^2$, Marangoni number; $\mathrm{We} = \sigma_1 Aa/\sigma_0$, modified Weber number; $\mathrm{Pr} = \nu^-/\chi^-$, Prandtl number; and $\eta(t) = (\sigma_1 A)^{-1}\rho^- ag\left(\frac{a^2}{\nu^-}t\right)$, dimensionless mass–force density;

3. Let us assumed that Ma and $\mathrm{Bo} = \sup|(\rho^0 - 1)\eta(t)|$ (analog of Bond's number) are much less than 1. For fixed physical parameters of liquids these conditions are realized if the quantities $a^2 A$ and $A^{-1}\sup|g(t)|$ are sufficiently small.* Expanding formally the functions $\mathbf{v}$, p, T in a series in Ma, we obtain for the first approximation the problem (2.1)-(2.4) with Ma = 0, which has an exact solution with a spherical interface $\Gamma_t \equiv \{|\mathbf{x}| = 1\}$. In this case, $V_n = 0$ and $K = -2$.

Let (r, φ, θ) be spherical coordinates, i.e.,

$$x_1 = r\cos\varphi\sin\theta,\ x_2 = r\sin\varphi\sin\theta,\ x_3 = r\cos\theta.$$

We shall seek a solution under the assumption of axial symmetry. We introduce the stream function $\psi(r, \theta, t)$ by the equations

$$v_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta},\ v_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r},$$

Stokes' system

$$\partial\mathbf{v}/\partial t = -\rho^{-1}\nabla p + \nu\nabla^2\mathbf{v}$$

then assumes the form

$$\begin{gathered} \frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial\xi}\left\{\nu E^2\psi - \frac{\partial\psi}{\partial t}\right\}, \\ \frac{1}{\rho}\frac{\partial p}{\partial\xi} = -\frac{1}{1-\xi^2}\frac{\partial}{\partial r}\left\{\nu E^2\psi - \frac{\partial\psi}{\partial t}\right\}, \end{gathered}$$

where $E^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\xi^2}{r^2}\frac{\partial^2}{\partial\xi^2}$; $\xi = \cos\theta$. Correspondingly, the components of the stress tensor have the following form in terms of ψ:

$$P_{r\theta} = -\frac{\mu}{(1-\xi^2)^{1/2}}\left\{E^2\psi - 2r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right)\right\},$$

*For example, for an air bubble in silicone oil at 1410°C and in pure water at 15°C, Ma and Bo are less than 1, if $a^2 A$ does not exceed $7.2\cdot10^{-6}$ and $8.7\cdot10^{-4}$ deg·cm, respectively, while $aA^{-1}\sup|g(t)|$ does not exceed 0.17 and 0.15 $\mathrm{cm^3\cdot sec^{-2}\cdot deg^{-1}}$.

$$\frac{\partial}{\partial \xi} P_{rr} = \mu \frac{\partial}{\partial r}\left\{\frac{1}{1-\xi^2}\left[E^2\psi - \frac{1}{\nu}\frac{\partial \psi}{\partial t}\right] + \frac{2}{r^2}\frac{\partial^2 \psi}{\partial \xi^2}\right\}.$$

As a result there arises the problem for the functions ψ, T, and u:

$$E^2[\nu^0 E^2\psi - \psi_t] = 0,\ \Pr T_t = \chi^0 \Delta T \quad \text{for} \quad r < 1, \tag{3.1}$$
$$E^2[E^2\psi - \psi_t] = 0,\ \Pr T_t = \Delta T \quad \text{for} \quad r > 1;$$

$$\psi^+ = 0,\ \psi^- = 0,\ \psi_r^+ = \psi_r^-, \tag{3.2}$$
$$\mu^0(\psi_{rr} - 2\psi_r)^+ - (\psi_{rr} - 2\psi_r)^- = (1 - \xi^2)T_\xi,$$
$$\varkappa^0 T_r^+ = T_r^-,\ T^+ = T^- \quad \text{at } r = 1;$$

$$\psi_r/r \to u(1 - \xi^2),\ \psi_\xi/r^2 \to -u\xi \quad \text{as} \quad r \to \infty; \tag{3.3}$$

$$\psi = 0,\ T = r\xi,\ u = 0 \quad \text{at} \quad t = 0; \tag{3.4}$$

$$(\rho^0 - 1)(u_t - \eta) + \mu^0\left\{\frac{E^2\psi - \nu^{0^{-1}}\psi_t}{1-\xi^2} + \frac{2}{r^2}\psi_{\xi\xi}\right\}_r^+ - \left\{\frac{E^2\psi - \psi_t}{1-\xi^2} + \frac{2}{r^2}\psi_{\xi\xi}\right\}_r^- = 2T_\xi \quad \text{at } r = 1. \tag{3.5}$$

Here $\Delta = \frac{1}{r^2}\left\{\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial \xi}\left[(1-\xi^2)\frac{\partial}{\partial \xi}\right]\right\}$; the subscripts r, ξ, t denote partial derivatives with respect to the corresponding variables. Equation (3.5) arose after differentiation of the normal component of the dynamic condition with respect to ξ.

4. The solution of the problem (3.1)-(3.5) is given by

$$\psi(r, \xi, t) = rf(r, t)(1 - \xi^2),\ T(r, \xi, t) = \Theta(r, t)\xi.$$

Let $u^*(s)$, $f^*(r, s)$, $\Theta^*(r, s)$ be the Laplace transforms of the functions $u(t)$, $f(r, t)$, $\Theta(r, t)$. Then, taking into account the initial conditions (3.4), we obtained a problem for u^*, f^*, Θ^*:

$$L^2[\nu^0 L^2 f^* - sf^*] = 0,\ \chi^0 L^2\Theta^* = \Pr[s\Theta^* - r] \quad \text{for} \quad r < 1, \tag{4.1}$$
$$L^2[L^2 f^* - sf^*] = 0,\ L^2\Theta^* = \Pr[s\Theta^* - r] \quad \text{for} \quad r > 1;$$

$$f^{*+} = 0,\ f^{*-} = 0,\ f_r^{*+} = f_r^{*-},\ \mu^0 f_{rr}^{*+} - f_{rr}^{*-} = \Theta^*, \tag{4.2}$$
$$\varkappa^0\Theta_r^{*+} = \Theta_r^{*-},\ \Theta^{*+} = \Theta^{*-} \quad \text{at} \quad r = 1;$$

$$f_r^* \to u^*/2,\ f^*/r \to u^*/2 \quad \text{at} \quad r \to \infty; \tag{4.3}$$

$$(1 - \rho^0)(su^* - \eta^*) + \{f_{rrr}^* + f_{rr}^* - (s + 6)f_r^*\}^- \tag{4.4}$$
$$= \mu^0\{f_{rrr}^* + f_{rr}^* - (s/\nu^0 + 6)f_r^*\}^+ \quad \text{at} \quad r = 1,$$

where $L^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{2}{r^2}$. From the integral identity

$$\int_0^1 (L^2\omega) r^3 dr = r^2(r\omega_r - \omega)_0^1$$

with the function $\omega = L^2 f^* - (s/\nu^0)f^*$ it is easily established that the right side of Eq. (4.4) is equal to zero. Thus the condition (4.4) simplifies to the following:

$$(1 - \rho^0)(su^* - \eta^*) + \{f_{rrr}^* + f_{rr}^* - (s + 6)f_r^*\}^- = 0 \quad \text{at} \quad r = 1. \tag{4.5}$$

5. Equations (4.1), taking into account the conditions that the velocity and temperature fields are bounded at r = 0 and the conditions (4.3), assume the following solution:

$$f^*(r, s) = C_1 F(\sqrt{s/\nu^0}r) + C_2 r,$$
$$\Theta^*(r, s) = r/s + C_3 F(\sqrt{s\Pr/\chi^0}r) \quad \text{for} \quad r < 1,$$
$$f^*(r, s) = u^*(s)r/2 + C_4 G(\sqrt{s}r) + C_5/r^2,$$
$$\Theta^*(r, s) = r/s + C_6 G(\sqrt{s\Pr}r) \quad \text{for} \quad r > 1,$$

where $F(z) = (\operatorname{sh} z/z)'$; $G(z) = (e^{-z}/z)'$; $(\cdot)' = d/dz$. The functions $C_1(s)$, ..., $C_6(s)$ are determined from the six equations (4.2), while $u^*(s)$ is determined from Eq. (4.5). As a result we obtain

$$f^*(r,s) = \frac{\Theta^*(1,s) + 3(1+\sqrt{s})\,u^*(s)/2}{3+\sqrt{s}+\mu^0 H(\alpha)} \frac{F(\alpha r) - F(\alpha)\,r}{\alpha F'(\alpha) - F(\alpha)}, \quad r < 1,$$

$$f^*(r,s) = \frac{\Theta^*(1,s) - 3[2+\mu^0 H(\alpha)]\,u^*(s)/2}{3+\sqrt{s}+\mu^0 H(\alpha)} e^{\sqrt{s}} \times$$

$$\times \left[G(\sqrt{s}r) - \frac{G(\sqrt{s})}{r^2}\right] + \frac{1}{2} u^*(s)\left(r - \frac{1}{r^2}\right), \quad r > 1,$$

$$\Theta^*(1,s) = \frac{1}{s}\left\{1 + (1-\varkappa^0)\left[\varkappa^0 \frac{\beta F'(\beta)}{F(\beta)} - \frac{\gamma G'(\gamma)}{G(\gamma)}\right]^{-1}\right\};$$

$$u^*(s) = \frac{C^*(s)\,\Theta^*(1,s) + (\rho^0 - 1)\,\eta^*(s)}{(1/2+\rho^0)\,s + B^*(s)}. \tag{5.1}$$

Here

$$H(z) = \frac{z^2 F''(z)}{zF'(z) - F(z)} = \frac{z(z^2+6) - 3(z^2+2)\operatorname{th} z}{(z^2+3)\operatorname{th} z - 3z};$$

$$B^*(s) = \frac{3}{2}[2 + \mu^0 H(\alpha)]\,C^*(s); \quad C^*(s) = \frac{3(1+\sqrt{s})}{3+\sqrt{s}+\mu^0 H(\alpha)};$$

$$\alpha = \sqrt{s/\nu^0}; \quad \beta = \sqrt{s\Pr/\chi^0}; \quad \gamma = \sqrt{s\Pr}.$$

The following asymptotic formulas hold:

$$H(z) = 3 + O(z^2), \quad z \to 0; \quad H(z) = z + O(1/z), \quad z \to +\infty.$$

From the asymptotic forms of $B^*(s)$, $C^*(s)$ in the limit $s \to +\infty$ it follows that the original functions $B(t)$ and $C(t)$ are generalized functions at $t = 0$. The transforms are therefore naturally represented as

$$B^*(s) = B^*(0) + sb^*(s), \quad C^*(s) = C^*(\infty) + c^*(s),$$

where $B^*(0) = 3(2 + 3\mu^0)/[2(1 + \mu^0)]$; $C^*(\infty) = 3/(1 + \rho^0\sqrt{\nu^0})$, and the original functions $b(t)$ and $c(t)$ are ordinary functions with the following asymptotic forms in the limit $t \to 0$

$$b(t) = \frac{9\rho^0\sqrt{\nu^0}}{2(1+\rho^0\sqrt{\nu^0})} \frac{1}{\sqrt{\pi t}} + O(1),$$

$$c(t) = -\frac{3(2-\rho^0\sqrt{\nu^0})}{(1+\rho^0\sqrt{\nu^0})^2} \frac{1}{\sqrt{\pi t}} + O(1),$$

while in the limit $t \to \infty$

$$b(t) = \frac{1}{2}\left(\frac{2+3\mu^0}{1+\mu^0}\right)^2 \frac{1}{\sqrt{\pi t}} + O(t^{-3/2}),$$

$$c(t) = -\frac{2+3\mu^0}{6(1+\mu^0)^2} \frac{1}{\sqrt{\pi t^3}} + O(t^{-5/2}).$$

As a result, the formula (5.1) leads to the following integrodifferential equation for $u(t)$

$$(1/2 + \rho^0)\,u'(t) + \int_0^t b(t-t_1)\,u'(t_1)\,dt_1 + \tag{5.2}$$

$$+ \frac{3(2+3\mu^0)}{2(1+\mu^0)} u(t) = Z(t) + (\rho^0 - 1)\,\eta(t),$$

where

$$Z(t)=\frac{3\Theta(1,t)}{1+\rho^0\sqrt{\nu^0}}+\int_0^t c(t-t_1)\,\Theta(1,t_1)\,dt_1.$$

6. Suppose that the function $\eta(t)$ has a limit as $t\to\infty$. Then, because of the equalities

$$\lim_{t\to\infty}\Theta(1,t)=\lim_{s\to0}s\Theta^*(1,s)=\frac{3}{2+\varkappa^0}$$

from (5.1) or (5.2) we find a formula for the limiting velocity

$$\lim_{t\to\infty}u(t)=\frac{2}{(2+\varkappa^0)(2+3\mu^0)}+\frac{2(1+\mu^0)}{3(2+3\mu^0)}(\rho^0-1)\lim_{t\to\infty}\eta(t).$$

The first term coincides with the thermocapillary drift velocity of the drop in the stationary case, obtained in [4]; the second term coincides with the rise velocity of the drop under the action of buoyancy force: represented by the Hadamard–Rybchinskii formula.

In an analogous manner we determine from (5.1) or (5.2) the initial acceleration of the drop:

$$(1/2+\rho^0)\,u'(0)=\frac{3}{1+\rho^0\sqrt{\nu^0}}+(\rho^0-1)\,\eta(0).$$

In addition, these equations enable finding the asymptotic expansion of $u(t)$ with integer powers of $\sqrt{t}$ in the limits $t\to0$ and $t\to\infty$.

In dimensional variables Eq. (5.2) can be put into the form of Newton's equation for the drop

$$(4/3)\pi a^3\rho^+\mathbf{u}'(t)=\mathbf{F}_M+\mathbf{F}_B+\mathbf{F}_S+\mathbf{F}_T+\mathbf{F}_A,$$

where $\mathbf{F}_M$ is the force generated by the effect of augmented masses; $\mathbf{F}_B$ is the analog of Bass's force; $\mathbf{F}_S$ is Stokes's force; $\mathbf{F}_T$ is the thermocapillary force; and $\mathbf{F}_A$ is the buoyancy force:

$$\mathbf{F}_M=-\frac{2}{3}\pi a^3\rho^-\mathbf{u}'(t),\ \mathbf{F}_B=-\frac{4}{3}\pi a\mu^-\int_0^t b\left(\frac{t-t_1}{a^2/\nu^-}\right)\mathbf{u}'(t_1)\,dt_1,$$

$$\mathbf{F}_S=-2\pi a\mu^-\frac{3\mu^++2\mu^-}{\mu^++\mu^-}\mathbf{u}(t),\ \mathbf{F}_T=-\frac{4}{3}\pi a^2\frac{d\sigma}{dT}Z\left(\frac{\nu^-}{a^2}t\right)\mathbf{A},$$

$$\mathbf{F}_A=\frac{4}{3}\pi a^3(\rho^+-\rho^-)\,\mathbf{g}(t).$$

If $\mu^0\to\infty$, then the thermocapillary force vanishes, and $\mathbf{F}_B$ transforms into the Bass force, arising when a solid sphere moves in the liquid; here

$$b^*(s)=9/(2\sqrt{s}),\ b(t)=9/(2\sqrt{\pi t}).$$

In the other limiting case $\mu^0=0$ (drift of a gas bubble)

$$b^*(s)=\frac{6}{\sqrt{s}(3+\sqrt{s})},\ b(t)=6e^{9t}\operatorname{erfc}(3\sqrt{t}),$$

$$c^*(s)=-\frac{6}{3+\sqrt{s}},\ c(t)=6\left\{3e^{9t}\operatorname{erfc}(3\sqrt{t})-\frac{1}{\sqrt{\pi t}}\right\},$$

where $\operatorname{erfc} z=\frac{2}{\sqrt{\pi}}\int_z^\infty e^{-z^2}dz$ is the complementary probability integral. Since the derivative $b'(t)=3c(t)$ is integrable on $(0,\infty)$, after integration by parts Eq. (5.2) assumes the form

$$(1/2+\rho^0)\,u'(t)+9u(t)+3\int_0^t c(t-t_1)\,u(t_1)\,dt_1=3\Theta(1,t)+\int_0^t c(t-t_1)\,\Theta(1,t_1)\,dt_1+(\rho^0-1)\,\eta(t).$$

The last equation can be reduced to a third-order differential equation for u(t). Indeed, the formula (5.1) with $\mu^0 = 0$ can be written as

$$[(1/2+\rho^0)s(\sqrt{s}+3)+9(\sqrt{s}+1)]u^*(s)=3(\sqrt{s}+1)\Theta^*(1,s)+(\rho^0-1)(\sqrt{s}+3)\eta^*(s)\equiv h^*(s).$$

Multiplying the left and right sides of this equality by

$$R^*(s)=(1/2+\rho^0)s(\sqrt{s}-3)+9(\sqrt{s}-1)$$

and introducing a notation for the cubic polynomial

$$Q(s)=(1/2+\rho^0)^2s^2(s-9)+18(1/2+\rho^0)s(s-3)+81(s-1),$$

we obtain the differential equation

$$Q(d/dt)u(t)=f(t),$$

where f(t) is a generalized function with the transform $f^*(s) = R^*(s)h^*(s)$.

We can now separate the regular part in f(t) and the singular part at t = 0, which contains information on the boundary conditions for u(t). As a result, the transition from the generalized Cauchy problem to the classical problem is made in a standard manner.

When $\mu^0 = \infty$ the corresponding reduction proceeds to a second-order differential equation (see [5]). The formula (5.1) assumes the form

$$[(1/2+\rho^0)s+(9/2)(\sqrt{s}+1)]u^*(s)=(\rho^0-1)\eta^*(s)$$

and is regularized by the symbol

$$R^*(s)=(1/2+\rho^0)s-(9/2)(\sqrt{s}-1).$$

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